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Steady state thickener modelling from the compressive yield stress and hindered settling function

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Abstract

An algorithm has been developed to predict steady state thickener operation from fundamental material properties, properly accounting for compression of the suspension network structure within the sediment bed. The material properties include the compressive yield stress, $P_y(\phi)$, and the hindered settling function, $R(\phi)$. $P_y(\phi)$ reflects the suspension network strength as a function of solids volume fraction ϕ, while $R(\phi)$ is inversely related to the permeability. The required inputs to the model include $P_y(\phi)$ and $R(\phi)$ curve fits, thickener diameter as a function of height, solids density, liquid density and feed solids volume fraction. The model output is either solids throughput or solids flux as a function of underflow solids concentration, for a range of suspension bed heights. As a bonus, the solids residence time in the suspension bed can also be determined.

The algorithm involves prediction of the solids throughput versus underflow solids concentration in two parts; free settling (clarification) and compression within the suspension bed (thickening). The free settling prediction utilises an adaption of the simple Coe and Clevenger method, while prediction of compression in the bed is achieved through integration of a differential equation developed from the fundamental dewatering theory of Buscall and White. The limiting steady state solids flux is the minimum of the two predicted values for each underflow solids concentration and bed height.

In just minutes, this algorithm can produce tabulated and graphical results providing useful insights into the inter-relationship between solids throughput, bed height and underflow solids concentration. For steady state thickener operation, the outputs reveal three general modes of stable operation; permeability limited at high solids fluxes, compressibility and permeability dependant at intermediate solids fluxes and compressibility limited at very low solids fluxes. Knowledge of the conditions under which each of these modes is applicable enables process operators to understand the effect of variations in process conditions and assists in process optimisation.

Keywords: Steady state thickener modelling; Compressive yield stress; Hindered settling function; Dewatering; Shear yield stress; Compressive rheology

1. Introduction

Thickening is a process that occurs in any type of clarifier, washer or settler that concentrates solids via settling and formation of a network structure or bed. Many incremental improvements in the performance of industrial thickeners have been based on selecting conditions that produce desired properties in settling test behaviour. Examples include selecting conditions that produce the desired settling rate, final sediment solids concentration, viscosity or shear yield stress. Though well entrenched and loosely based on material properties, such as permeability, compressibility and shear rheology, this type of empirical method does not enable quantitative prediction of thickener performance.

Compressive yield stress and permeability have been established as fundamental physical properties that determine suspension dewaterability [1]. Experimental techniques have been developed to allow rapid and comprehensive dewaterability characterisation over a wide range of solids concentrations using batch settling, gravity permeation, centrifugation and pressure filtration tests [2–6].

Numerous authors [7–9] have presented fundamentally based equations and computational algorithms for predicting transient thickener performance. The outputs of these algorithms enable understanding of how long it takes for process

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variations to have their effect on process performance. The key disadvantage of these methods is their complexity, often resulting in significant computation time, enabling only a small number of conditions to be modelled. Prediction of steady state thickening performance is computationally simpler than the transient counterpart, thus enabling a wider range of conditions to be modelled. Again, numerous authors [2,3,10–12] have presented fundamentally based equations and algorithms to predict steady state thickener performance. Methods presented by Green [3], Landman et al. [11] and Landman and White [12] properly modelled consolidation in the suspension bed, but not sedimentation above the bed. The methodologies of Garrido et al. [10] and Usher [2] are mathematically consistent, but apply very different computation algorithms and present outputs differently. Garrido et al. [10] present outputs as solids concentration profiles in the bed as a function of volumetric throughput rather than solids throughput, which sometimes obscures understanding of how to optimise a process. This paper introduces the algorithm by Usher [2] for which a modelling tool has been developed to facilitate prediction using curve fits of experimentally determined dewaterability data with the fundamentally based mathematical theory of Buscall and White [1]. The technique predicts the steady state solids throughput for a given feed solids concentration and thickener dimensions as a function of suspension bed height and underflow solids concentration. Presenting performance prediction outputs in this manner enables improved understanding of how process variables influence thickener output.

2. Material properties

As a prelude to introducing the computational algorithm, it is important to understand the material properties involved. The gel point, ϕ_g, represents the solids concentration at which the suspension forms a continuously networked structure. The network strength is quantified in terms of the compressive yield stress, $P_y(\phi)$, which is, for a suspension at solids concentration ϕ, the maximum compressive stress that can be applied before irreversible yielding and dewatering to a higher solids concentration. $P_y(\phi)$ is zero for all solids concentrations below ϕ_g. The rate at which a material can be dewatered is quantified in terms of the hindered settling function, $R(\phi)$, which represents the resistance to flow through the suspension network structure and is inversely related to the traditional Darcian permeability $k_{Darcy}(\phi)$ according to the following equation [13]:

$$k_{\mathrm{Darcy}}(\phi) = \frac{\eta}{R(\phi)}\frac{1-\phi}{\phi} \tag{1}$$

The shear yield stress, $\tau_y(\phi)$, often termed the yield stress, represents the shear stress required for a suspension at solids concentration ϕ to irreversibly yield and flow. Though $\tau_y(\phi)$ is not utilised by this dewatering model, it does provide a means of evaluating whether the output suspension can be raked and pumped. The relationship between $\tau_y(\phi)$ and $P_y(\phi)$ has been measured for a number of systems and generally follows a fixed ratio up to a critical concentration [14,15].

2.1. Material property characterisation

A detailed protocol for the quantification of the dewatering parameters to be used as inputs to the dewatering model, has been established. The experimental techniques employed are generic to the assessment of dewatering performance in a range of industries. Techniques of relevance include batch settling tests [2–4] for permeability analysis at low solids and for compressibility analysis at solids near the gel point and stepwise pressure filtration [5,6] for permeability and compressibility analysis at higher solids. Other techniques including gravity permeation [2] and centrifugation [3] have also been developed. The shear yield stress can typically be characterised over a range of solids concentrations using a vane rheometer [19,20] or slump test [21].

Once obtained, the experimental data can be fitted with curves such as those proposed by Landman et al. [11]. The most commonly utilised functional forms are given by:

$$P_y(\phi) = k\left(\left(\frac{\phi}{\phi_g}\right)^n - 1\right) \quad \text{for } \phi > \phi_g, \tag{2}$$

$$R(\phi) = w(1-\phi)^m, \tag{3}$$

where $P_y(\phi) = 0$ for $\phi \le \phi_g$ and k, n, w and m are empirical fitting parameters. Unfortunately, these functional forms have been found to be too rigid and thus unable to provide an adequate fit to experimental data that spans a wide range of solids concentrations [2]. Two alternatives that have been shown to provide a better fit are:

$$P_y(\phi) = \left(1 - \left(\frac{\phi_g}{\phi}\right)^{p_m}\right) e^{p_a\phi^{p_n}+p_b} \quad \text{for } \phi > \phi_g, \tag{4}$$

$$R(\phi) = r_a(\phi - r_g)^{r_n} + r_b, \tag{5}$$

where $P_y(\phi) = 0$ for $\phi \le \phi_g$ and p_a, p_b, p_m, p_n, r_a, r_b, r_g and r_n are empirical fitting parameters. Curve fits using Eqs. (4) and (5) are presented in Figs. 1 and 2. Other potential alternatives include interpolation functions or composite functions which split the curve into a number of domains, each with its own parameter set.

3. Thickener dewatering performance prediction

A pseudo two-dimensional thickener modelling capability has been developed to quantify the role of flocculants in dewatering. This involves a steady state thickening calculation algorithm [2] which combines free settling rate theory [16] with suspension bed consolidation theory [3]. This modelling tool can be applied for both flat bottomed thickeners and converging base thickeners through the use of a shape factor to account for cross sectional area variations [3]. The assump-

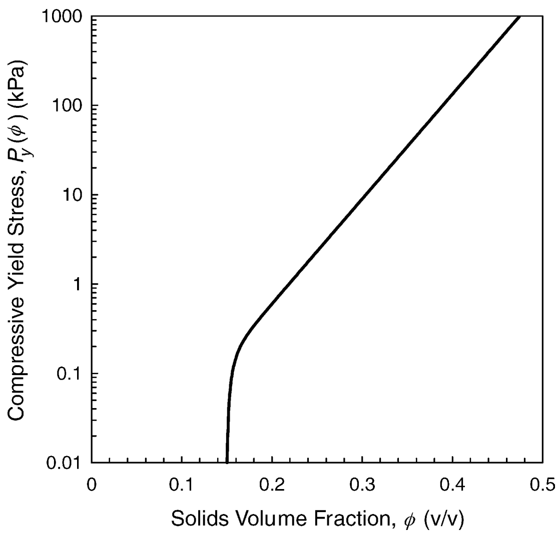

Fig. 1. Example of compressive yield stress, $P_y(\phi)$, curve fit using the asymptote and exponential-power law functional form given by Eq. (4) with parameter values $\phi_g = 0.15$, $p_a = 27$, $p_b = 1$, $p_m = 20$ and $p_n = 1$.

tions, inputs and calculation algorithm are described below. The theory has been converted into code, enabling timely simulation of steady state thickening from dewaterability data. The output includes steady state solids flux predictions for a range of underflow solids concentrations and suspension bed heights.

3.1. Assumptions

As with all process models, there are a number of assumptions and these should be kept in mind when utilising the model output. The assumptions are as follows:

- The model used is one-dimensional.

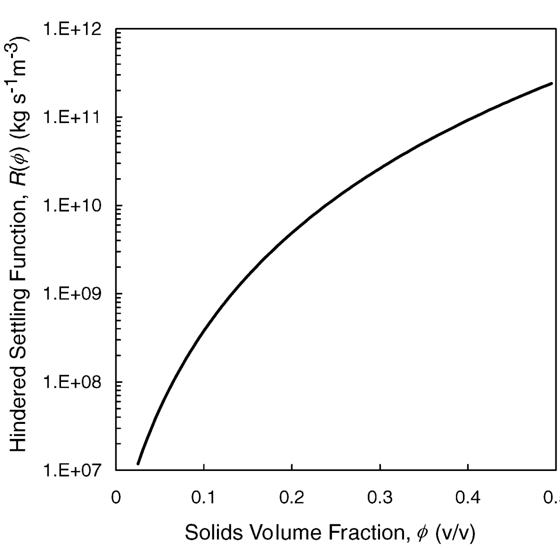

Fig. 2. Example of hindered settling function, $R(\phi)$, curve fit using the functional form given by Eq. (5) with parameter values $r_a = 5 \times 10^{12}$, $r_b = 0$, $r_g = -0.05$ and $r_n = 5$.

It is converted to a two-dimensional model by the use of a shape factor, so the model does not account for short circuiting and mixing. This implies that only vertical dewatering is predicted and horizontal flow of liquor is ignored. As a result, non-isotropic permeability variations are not taken into account.
- The model assumes line settling.
  This implies that settling rate and permeability are functions of solids volume fraction and all solid particles at the same height settle at the same rate, with no particle size segregation.
- The model does not account for shearing.
  Shear forces in the thickener, caused by the action of rakes and shear rods, are expected to improve thickening. However wall friction which can slow consolidation is also not included.
- The model assumes that no solids exit via the overflow.
- The model assumes steady state thickener operation.

3.2. Inputs

The inputs required for the steady state thickener model include:

- A compressive yield stress, $P_y(\phi)$, curve fit.
- A hindered settling function data, $R(\phi)$, curve fit.
- The thickener dimensions, including diameter as a function of height, $d(z)$, such as that shown in Fig. 3.
- The feed solids concentration, ϕ_0.
- Solid and liquor densities, ρ_{sol} and ρ_{liq}.

All modelling results presented in this paper utilise the $P_y(\phi)$ and $R(\phi)$ curve fits specified in Figs. 1 and 2, converging base thickener dimensions such that the diameter is 1 m at the base ($h_b = 0$) and $d_{max} = 40$ m for $h_b \geq 5$ m, a feed solids concentration $\phi_0 = 0.05$, a solids density $\rho_{sol} = 3200$ kg m and a liquid density $\rho_{liq} = 1000$ kg m^{-3}.

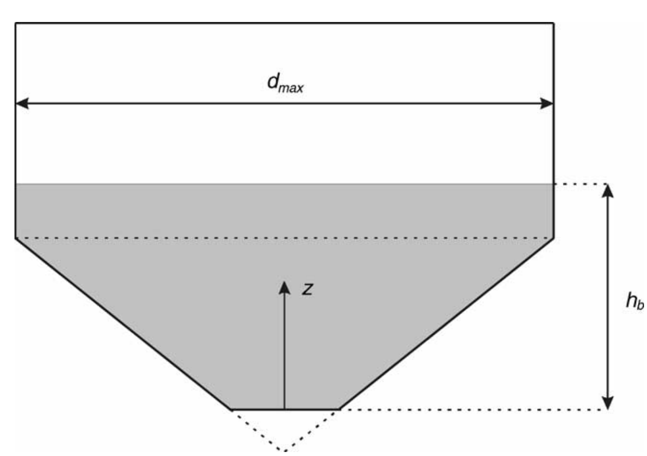

Fig. 3. Schematic of a converging base thickener, with truncated cone, showing height in the thickener, z, the suspension bed height, h_b, and the maximum thickener diameter, d_{max}.

3.3. Calculations

Steady state thickener modelling involves prediction of the solids throughput as a function of underflow solids concentration and suspension bed height in two parts. The first part deals with free settling (generally called clarification) while the second part considers compression in the suspension bed (thickening). The free settling and compression predictions are combined to predict the steady state solids flux by taking the minimum predicted solids flux for each underflow solids concentration. It is important to note that in all of the following equations, the solids flux, q, is defined as a volume of solids per unit time per thickener cross sectional area, with SI units of $\mathrm{kg\,s^{-1}\,m^{-2}}$. The cross sectional area referred to is that of the vessel at the top of the suspension bed, where the height $z = h_b$, and solids concentration $\phi = \phi_g$. However, to adhere to industry conventions, all graphs of solids flux are presented in tonnes of solids per hour per square meter, while all graphs of solids throughput are presented in tonnes of solids per hour.

3.4. Free settling

Coe and Clevenger [17] proposed that the settling rate is a function of solids concentration as long as no mechanical support is contributed from layers of suspension below [16]. They called this condition free settling. Indeed, using the theory of Buscall and White [1], the free settling rate, $u_{fs}(\phi)$, of a suspension in the absence of a compressive yield stress influence is predicted to be a function of the solids volume fraction, ϕ, according to the following:

$$u_{fs}(\phi) = \frac{\Delta\rho g(1-\phi)^2}{R(\phi)}, \tag{6}$$

where $\Delta\rho = \rho_{sol} - \rho_{liq}$, is the solid liquid density difference and $g = 9.8\,\mathrm{m\,s^{-2}}$ is the gravitational constant. The traditional Coe and Clevenger method suggests that this settling rate be used in a material balance to determine the thickener steady state solids flux (for the maximum thickener cross sectional area), q, for suspension at any solids concentration, ϕ, and for a given underflow solids concentration, ϕ_u [16].

$$q = \frac{u_{fs}(\phi)}{1/\phi - 1/\phi_u} \tag{7}$$

For a given underflow solids concentration, ϕ_u, and suspension bed height, h_b, the maximum thickener capacity possible in free settling, q_{fs}, will be the minimum value of solids flux, q, obtained by applying the material balance for all solids concentrations, ϕ, ranging from ϕ_0 to ϕ_u. The theory and methodology presented here is consistent with that presented by other recent workers, such as Bustos et al. [18], and an analogous algorithm has been presented by Garrido et al. [10].

3.5. Compression

The underflow solids concentration predicted by free settling is often not achieved because suspension compressibility also limits the underflow solids concentration. Traditional methods of predicting thickening behaviour assume an incompressible suspension bed with a constant solids concentration, which is fundamentally flawed when a material exhibits a compressive yield stress over a range of solids concentrations. The technique used in this analysis involves integration of a pseudo two-dimensional differential equation that has been developed from fundamental dewaterability theory [3,11]:

$$\frac{d\phi(z)}{dz} = \frac{[R(\phi(z))/(1-\phi(z))^2][q/\alpha(z)] \times [1-\phi(z)/\phi_u] - \Delta\rho g\phi(z)}{dP_y(\phi(z))/d\phi(z)}, \tag{8}$$

where a thickener shape factor, $\alpha(z)$, defined as,

$$\alpha(z) = \left(\frac{d(z)}{d_{max}}\right)^2, \tag{9}$$

accounts for the cross sectional area variations with height in the thickener, z. The definition of $\alpha(z)$ is such that $\alpha = 1$ when the thickener diameter, $d(z)$, is at its maximum, d_{max}, and the cross sectional area is also maximised. The pseudo two-dimensional differential equation relates the change in solids concentration with height in the thickener, $d\phi(z)/dz$, for a given steady state solids flux, q, and underflow solids concentration, ϕ_u.

At steady state, the solids concentration is equal to the gel point, ϕ_g, at the top of the suspension bed, $z = h_b$, and is equal to the given underflow solids concentration, ϕ_u, at the base of the thickener, $z = 0$.

To determine the solids flux, q, required to produce a steady state suspension bed height, h_b, and underflow solids concentration, ϕ_u, the differential equation is integrated from the bottom to the top of the suspension bed, $z = 0$ to h_b, subject to the boundary condition,

$$\phi(0) = \phi_u, \tag{10}$$

using an initial guess for, q. The value of q is bounded between 0 and the permeability limit determined in the free settling calculations for the given ϕ_u.

The solids flux guess is iteratively adjusted through repeated integrations, until the solids concentration at the top of the bed equals the gel point,

$$\phi(h_b) = \phi_g. \tag{11}$$

An alternative description of Eq. (10) can improve computational speed. The inverted differential equation, shown below,

$$\frac{dz(\phi)}{d\phi} = \frac{dP_y(\phi)/d\phi}{[R(\phi)/(1-\phi)^2][q/\alpha(z(\phi))][1-\phi/\phi_u] - \Delta\rho g\phi}, \tag{12}$$

relates the change in height with solids concentration in the thickener, $dz(\phi)/d\phi$, for a given steady state solids flux, q, and underflow solids concentration, ϕ_u. The differential equation is integrated from the bottom to the top of the suspension bed, $\phi = \phi_u$ to ϕ_g, subject to the boundary condition,

$$z(\phi_u) = 0, \tag{13}$$

using an initial guess for, q. The solids flux guess is iteratively adjusted through repeated integrations, until the solids concentration at height h_b equals the gel point,

$$z(\phi_g) = h_b. \tag{14}$$

The integration method is repeated for a number of underflow solids concentrations, ϕ_u, and suspension bed heights, h_b, to produce curves of q versus ϕ_u, for a number of h_b values generally ranging from 0.1 to 20 m.

3.6. Combining results

The free settling and compression predictions are combined to predict the limiting steady state solids flux by taking the minimum predicted solids flux for each underflow solids concentration as shown in Fig. 4. This approach is proposed by Coe and Clevenger [17] for settling rate dependant thickening and it is a logical extension of this idea to apply this methodology for dewatering in the bed as well. The computational algorithm that was used to produce the results presented in this paper are provided in Appendix A.

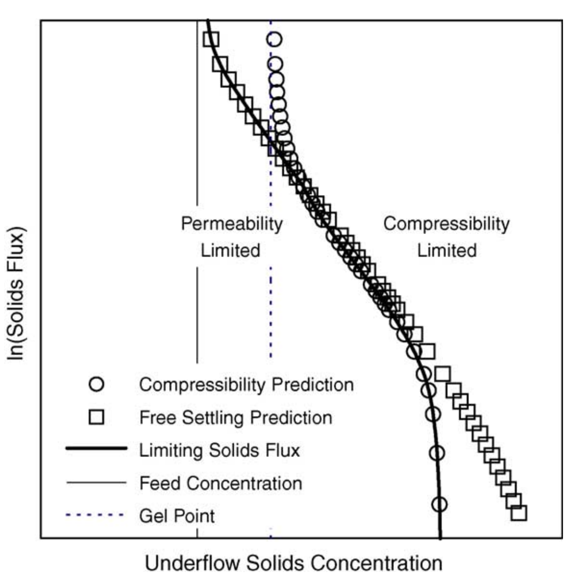

Fig. 4. Combining free settling and compression predictions to predict the limiting steady state solids flux.

4. Outputs – thickener dewatering performance prediction

4.1. Solids flux

An algorithm has been programmed in Mathematica code and also C-code to take the input data and determine the steady state solids flux for a range of underflow solids concentrations and suspension bed heights. The resultant output data, derived from the $P_y(\phi)$ and $R(\phi)$ inputs in Figs. 1 and 2, is presented in Fig. 5. The data illustrate the expected underflow solids achievable for a given solids flux through the thickener (flux is calculated from the solids throughput per unit cross sectional area of the vessel at the top of the suspension bed). The effect of different suspension bed heights on the predicted underflow solids concentrations is illustrated. The gel point and feed solids concentration are also illustrated. The operation of a thickener may be broken into two distinct regions, each of which will be discussed in turn.

4.2. Permeability limited operation

For moderate to high solids fluxes, >0.1 t h^{-1} m^{-2} in this example, thickener operation is permeability limited. This implies that the rate at which the solids are passed through the vessel is so fast that no transmission of compressive forces is achieved in the suspension bed. The limiting factor is the rate at which the liquid is able to escape from the solids network, which is dictated expressly by the permeability. Therefore, the model predicts that in permeability limited operation, the steady state underflow solids concentration is a function of solids flux alone, and not influenced by suspension bed height. Indirectly however, the suspension bed height may also influence the effectiveness of raking (not ac-

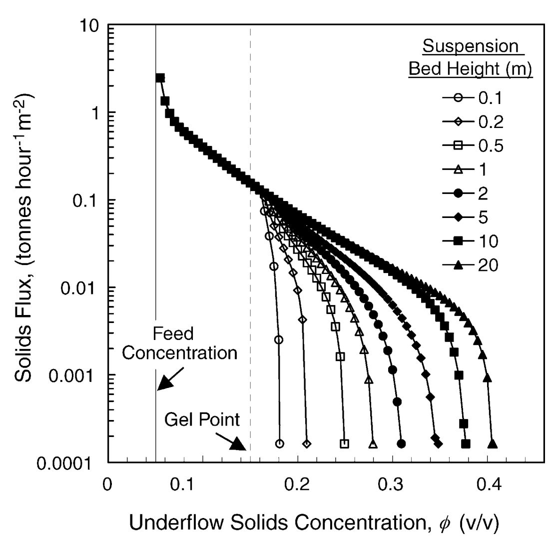

Fig. 5. Typical steady state converging base thickener model prediction of the solids flux as a function of underflow solids volume fraction for a range of suspension bed heights.

counted for in the model) that also influences the underflow solids concentration.

4.3. Compressibility limited operation

At low to intermediate solids fluxes, $<0.1\,\mathrm{t\,h^{-1}\,m^{-2}}$ in this example, the suspension bed height is observed to have an effect on the underflow solids concentration. In this region, the residence time of the solids in the suspension bed is long enough for compressive dewatering to occur. As such, the amount of compressive force transmitted by the network structure of the suspension bed is the dominant effect that governs the underflow solids concentration.

In the limit of zero solids flux, the suspension will settle to equilibrium and the underflow solids concentration will be a function of steady state bed height only, dictated by the compressive force generated and the compressive yield stress behaviour for the material in question.

4.4. Solids throughput

For a flat bottomed thickener, multiplying the solids flux by the cross sectional area of the thickener yields the solids throughput. Therefore, for a flat bottomed thickener, a plot of solids throughput ($\mathrm{t\,h^{-1}}$) versus underflow solids concentration has the same shape as its solids flux counterpart ($\mathrm{t\,h^{-1}\,m^{-2}}$), with the same suspension bed height effects, just with the y-axis re-scaled. In the permeability-limited region of operation, for solids throughput and solids flux, the underflow solids concentration is predicted to be insensitive to the suspension bed height. In this region of operation, the model suggests that the only options available for changing the underflow solids concentration are either to reduce the solids flux (not practical) or to improve the permeability of the material in the low solids region, below the gel point.

For a converging base thickener, solids throughputs are determined by multiplying the solids fluxes by the cross sectional area of the thickener at the applicable suspension bed heights. The converging base thickener solids flux curves shown in Fig. 5 have been replotted as solids throughput curves in Fig. 6. The separation of the curves shown in Fig. 6, for suspension bed heights up to 5 m, is due to variation in cross sectional area at the top of the suspension bed as the suspension bed height varies. For a converging base thickener, the cross sectional area increases with the height of the suspension bed. As a consequence, when the top of the suspension bed lies within the conical region, the solids throughput is a function of bed height even when the process is permeability limited. Therefore, for converging base thickeners, the solids throughput is a function of suspension bed height in situations where the solids flux is not. However, for higher suspension bed heights, where thickener diameter does not vary with suspension bed height, the bed height effect is only observed for low solids fluxes.

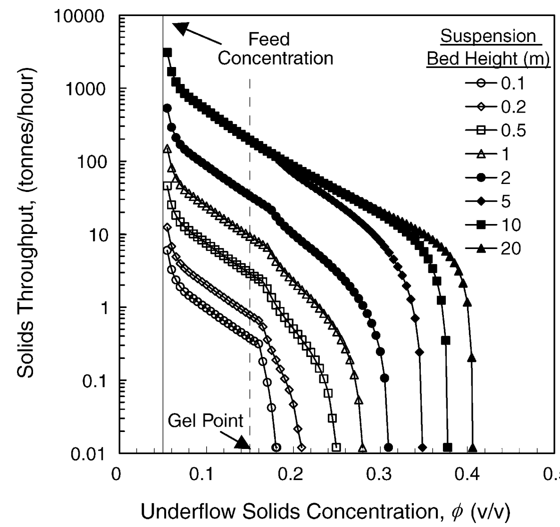

Fig. 6. Typical steady state converging base thickener model prediction of the solids throughput as a function of underflow solids volume fraction for a range of suspension bed heights.

4.5. Solids residence time

The steady state thickener model predictions may also be used to calculate the effective residence time of solids in the bed of a thickener, t_{res}, according to the following equation:

$$t_{\mathrm{res}} = \frac{1}{q}\int_0^{h_{\mathrm{b}}} \alpha(z)\phi(z)\,\mathrm{d}z. \qquad (15)$$

Fig. 7 shows a typical bed solids residence time versus underflow solids concentration for a thickener based on model predictions. The type of data presented in Fig. 7 gives insight into the interaction between underflow solids concentration and the existence of liquor stability issues and whether pre-

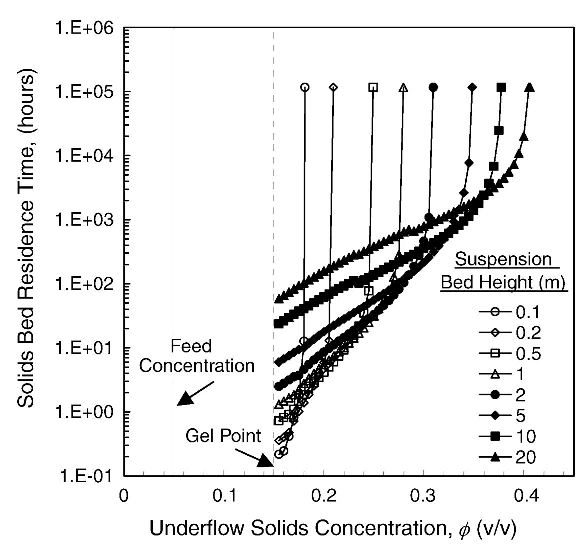

Fig. 7. Typical steady state thickener model prediction of the solids residence time in the suspension bed as a function of underflow solids volume fraction for a range of suspension bed heights.

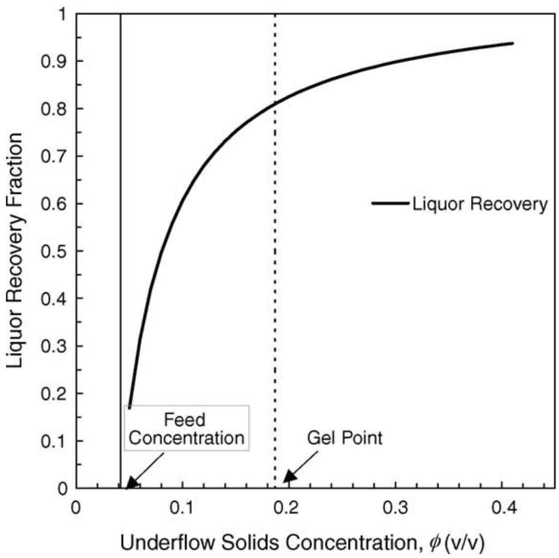

Fig. 8. Typical steady state thickener model prediction of liquor recovery fraction as a function of the underflow solids volume fraction.

cipitation or other time dependant behaviour may be expected to be a problem in a given system.

4.6. Fractional liquor recovery

A liquor material balance can be used to indicate the fractional liquor recovery, $f_{\rm lr}$, in a thickener (or any other dewatering operation for that matter) as a function of the underflow solids concentration. The fractional liquor recovery is defined as the liquor overflow flowrate divided by the total flowrate of liquor fed to the washer, assuming that all solids leave via the underflow and can be calculated from the following equation:

$$f_{\rm lr} = \frac{\phi_{\rm u} - \phi_0}{\phi_{\rm u}(1 - \phi_0)}. \qquad (16)$$

Fig. 8 shows the fractional liquor recovery as a function of feed solids concentration, for the thickening example being used. For underflow solids volume fractions less than 0.09 (corresponding to the condition of high overall solids flux) the liquor recovery is low, below 50%. This indicates an impractical region of operation due to the free settling rate of the feed material. Alternatively, for an underflow solids volume fraction of 0.30, liquor recovery is over 90% and there is limited potential for improvement, even with significantly higher underflow solids concentrations. The fractional liquor recovery puts underflow solids concentrations into context by giving an alternative measure of dewatering efficiency to underflow solids volume fraction.

5. Discussion

5.1. Comparison of predictions with process outputs

Comparison of industrial thickening process performance with steady state model predictions provides a good measure of the shortcomings of a predictive model. For a number of operating industrial thickeners processing flocculated red mud, the thickener dimensions, operating parameters and characterised material properties have been used as inputs to the steady state thickening algorithm by Usher [2]. The predictions of solids flux versus underflow solids concentration highlight that most thickener operation is limited by suspension permeability. At the actual underflow solids concentration, comparison of the predicted solids flux with the actual value has suggested that there is something in the thickener operation that effectively improves permeability by a factor ranging from 2 to 100 for various raked thickeners. This permeability enhancement, not accounted for in the model, has been attributed to raking and other shear processes which are the focus of a number of ongoing research projects.

5.2. Shear yield stress limited operation

Industrial observations suggest that thickening performance can often, but not always, be controlled by modifications to flocculant dose, bed height and rake speed. From an operational perspective, the maximum underflow solids concentrations achievable in many thickeners are limited by physical constraints. These constraints include an upper limit in the rake torque that can be safely applied and an upper limit of the rate at which the underflow pump can operate for the underflow shear rheology. The simplest measure of the rakeability and pumpability of a suspension is the shear yield stress. As a demonstrative example, many paste thickener raking arms and underflow pumps cannot handle suspensions with a shear yield stress greater than 200 Pa.

Fig. 9 shows a solids throughput versus underflow solids concentration prediction for a thickener operated with 5 m of bed height, as is presented in Fig. 6. The gel point and feed solids concentration are also illustrated, in addition to the solids concentrations at which the shear yield stress of

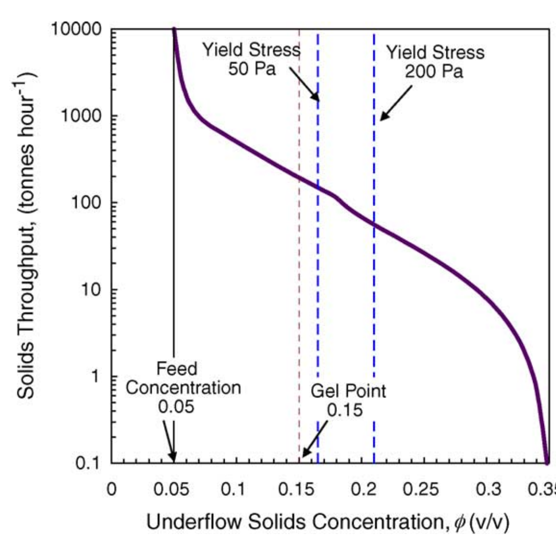

Fig. 9. Typical steady state thickener model prediction of the solids throughput as a function of underflow solids concentration for a bed height of 5 m.

the material is 50 and 200 Pa. Now, even though the thickener modelling prediction shown in Fig. 9 indicates that an underflow solids concentration greater than 0.21 is achievable, this is not likely to be the case in practice. As a process control measure, ensuring that the underflow solids concentration does not become too high can involve lowering the suspension bed height, slowing the rake arm or lowering the flocculant dose.

5.3. Optimisation

In terms of process optimisation, the aim is generally to maximize the solids throughput and underflow solids concentration while maintaining overflow clarity and underflow yield stress within preset bounds. Selecting the flocculation regime that will enable this optimisation often requires a compromise between the conditions that produce the highest permeability and those which produce the best compressibility. Alternatively, permeability curves may actually cross each other within the solids concentration range of interest. Neither of these optimisations is possible without a predictive thickener modelling tool.

A methodology has been developed to exploit the outputs of the thickener algorithm to enable quantitative optimisation of thickener performance. The methodology involves characterisation of material properties including permeability, compressibility and shear rheology for a range of test conditions such as flocculant dose. Thickener dewatering performance predictions are then produced for each test condition based on the characterisation results. Comparison of the results quantitatively predicts the effect of varying flocculant dose on thickener performance.

It should be stressed that the model predictions are subject to limitations relating to the idealised nature of the model. The absence of the ability to account for shear, dead-zones and unstable operation means that suggestions for process optimisation should be treated with caution. In addition, considering that the compressibility and permeability data cover up to five orders of magnitude, the outputs of the model are very sensitive to measured and curve fitted permeability values. As a result, the model predictions are not applicable to extremely accurate prediction of thickener performance, however, they do improve understanding of why certain underflow concentrations are achieved, the trends involved and how performance can be improved by adjustment of process variables. Process variables that can be examined include:

- The influence of feed solids concentration, solids flux and thickener dimensions.
- Whether improvements in compressibility or permeability will be most beneficial.
- The sensitivity of underflow solids concentration to process variations.
- The influence of type of flocculant and flocculation conditions.
- The impact of upstream processing options such as coarse/fine separation or blending.
- The impact on downstream options such as management of tailings impoundments and water recovery.
- The response in terms of water recovery and underflow density to all of the suggestions above.

6. Conclusions

A useful steady state thickening model calculation algorithm has been developed by combining free settling rate theory with suspension bed consolidation theory. The algorithm uses fundamental compressive yield stress, hindered settling function and thickener geometry data as inputs. Presentation of model outputs as solids flux and solids throughput versus underflow solids concentration has been shown to enable improved understanding of how steady state performance can be affected by the adjustment of process variables. Consequently, the role of flocculants, raking and other process variables in industrial thickening operations can be quantitatively predicted, improving the potential for process optimisation.

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Appendix A. Computational algorithm

The computational algorithm that was applied to predict steady state thickening performance is described below:

1. A list of underflow solids volume fractions, ϕ_u, was created with values bounded between the initial solids volume fraction, ϕ_0 and a high value such as 0.64 (e.g. If $\phi_0 = 0.05$, then the list may be $\{0.06, 0.07, \ldots, 0.63, 0.64\}$).
2. For each underflow solids concentration, ϕ_u, the minimum free settling flux, q_{fs}, was determined by application of Eq. (7) for all solids concentrations, ϕ, ranging from ϕ_0 to ϕ_u as described in the Free Settling theory.
3. A list of bed heights, h_b, was created with optimum output resolution achieved by increasing values in a roughly exponential manner. For example $\{0.1, 0.2, 0.5, 1, 2, 5, 10, 20\}$.
4. The relevant bed height bounds z_{equil} and z_{free} were calculated for each specified underflow solids concentration, ϕ_u, where $\phi_u > \phi_g$. The minimum bed height required to achieve ϕ_u, defined as $z_{equil} = z(\phi_g)$, was determined by integration of Eq. (12) with $q = 0$. The bed height beyond which thickener operation is permeability limited is defined as $z_{free} = z(\phi_g)$. For a cylindrical vessel ($\alpha(z) = 1$), z_{free} was determined by integration of Eq. (12)

with $q = 0.99q_{fs}$ (Note that if $q \geq q_{fs}$, then Eq. (12) cannot be solved to uniquely satisfy the boundary conditions). When the thickener has a converging base ($\alpha(z) < 1$), it is more difficult to avoid unstable integrations. In this case, the value of z_{free} was initially guessed as 0, and then iteratively determined as $z_{free} = z(\phi_g)$, by integration of Eq. (12) with $q = \alpha(z_{free})q_{fs}$. Eq. (12) was iteratively solved until the variation of z_{free} between iterations was insignificant (e.g. $<10^{-8}$).

5. For each bed height, a list of q values was created corresponding to the list of ϕ_u values such that; if $\phi_u < \phi_g$ then $q = q_{fs}$. Else, if $\phi_u \leq \phi_g$ then;
   a. if $h_b < z_{equil}$, ϕ_u was unattainable and there was no corresponding q value,
   b. if $z_{equil} \leq h_b < z_{free}$, q was determined via a shooting method described below and
   c. if $h_b \geq z_{free}$, then $q = \alpha(z_{free})q_{fs}$.
6. To convert q values to solids fluxes (m^3 solids s^{-1} m^{-2} at height h_b), q was multiplied by the ratio of the maximum cross sectional area to that at height h_b given by $d_{max}^2/d(h_b)^2$.
7. To convert q values to solids throughputs (m^3 solids s^{-1}), q was multiplied by the maximum cross sectional area given by $\pi d_{max}^2/4$.

The shooting method for determining q, given ϕ_u and h_b, involved initially setting the left and right bounds of q such that $q_l = 0$ and $q_r = \alpha(z_{free})q_{fs}$. Using the midpoint between q_l and q_r as a guess for q, $q_{guess} = (q_l + q_r)/2$, Eq. (12) was solved to determine $z(\phi_g)$. If $z(\phi_g) > h_b$, then $q_r = q_{guess}$, else $q_l = q_{guess}$. The process was iteratively repeated until the required accuracy was achieved (e.g. $|q_r - q_l| < 10^{-8}$).

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